Journal of Engineering Physics and Thermophysics, Vol. 83, No. 1, 2010

MATHEMATICAL MODELING OF THERMAL OPERATING REGIMES OF ELECTRIC RESISTANCE FURNACES

P. S. Grinchuk UDC 536.33

A physicomathematical model making it possible to calculate thermal regimes of electric resistance furnaces has been proposed. The model is suitable for description of linings and heated products of different types. It includes, as components, the following models: those of thermal-radiation transfer in the furnace's workspace, of nonstationary heating of target products, of nonstationary heating of the furnace lining, and of external heat exchange of the furnace's enclosing structures. The distinctive features of a numerical method for solution of the proposed model are discussed. An example of calculation of a chamber electric resistance furnace for the cases where it is lined with fireclay brick and lightweight fibrous materials is discussed. It is shown that replacement of the lining by a fibrous one improves the thermal operating efficiency for this type of furnace 2–2.5 times.

Keywords: *electric resistance furnace, thermal regime, mathematical modeling.*

The current development of the industrial potential of the Republic of Belarus is largely associated with the metal working of metallurgical products (steel and cast-iron rolled products) produced in the country and imported from abroad and with the creation, on this basis, of high-tech machines and mechanisms. Heat-treatment operations (annealing, tempering, quenching), which make it possible to obtain the required mechanical properties of metal structures, to relieve internal mechanical stresses in the metal, etc., are an important stage of metal working. Furnace heating, gas-flame or electric, is mainly used for heat treatment at the Republic's industrial enterprises.

Electric heating possesses a number of important advantages that cannot, in principle, be realized in gas-flame heating. Electric heating primarily enables one to obtain a high temperature uniformity in the furnace's workspace, which is extremely important for the quality of the end product in some cases. For example, whereas a uniformity of $\pm 20^{\circ}$C is assumed to be a high index for a gas-flame furnace at a workspace temperature of 900°C, a uniformity of $\pm 5^{\circ}$C or even lower is quite attainable in electric heating. Also, electric furnaces, by virtue of a number of their distinctive features, are of higher efficiency compared to gas-flame furnaces. The only drawback of electric furnaces is the high cost of primary electric energy, which is closely associated with the comparatively low efficiency of electric power stations converting thermal power to electric power.

The issues of thermal calculation of structures and technological parameters of electric resistance furnaces were the focus of quite an ample amount of scientific literature [1–4]. However, the appearance of new heat-insulating materials in recent times and reaching a qualitatively new level by the hardware of automatic control systems have led to the necessity of changing substantially the designing of industrial electric resistance furnaces. One basic issue of designing is the correct selection of the installed furnace capacity (power) ensuring required technological regimes of operation. If the installed capacity is insufficient, the furnace may simply not reach a prescribed temperature regime. The long time of reaching the regime will also lead to an additional heat loss through enclosing structures. The excess capacity gives rise to the overexpenditure of both the material of heaters and the number of fastening elements. Installation of a large capacity involves, as a rule, additional structural problems: a larger number of heaters must be arranged on the same area, sometimes immediately adjacent to each other.

The issue of correct selection of the capacity can, in principle, be solved in two ways. The first (experimental) method is to create several furnaces and to select an experimentally optimum capacity. This method is expensive

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A. V. Luikov Heat and Mass Transfer Institute, National Academy of Sciences of Belarus, 15 P. Brovka Str., Minsk, 220072, Belarus; email: gps@hmti.ac.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 83, No. 1, pp. 28–37, January–February, 2010. Original article submitted October 9, 2009.

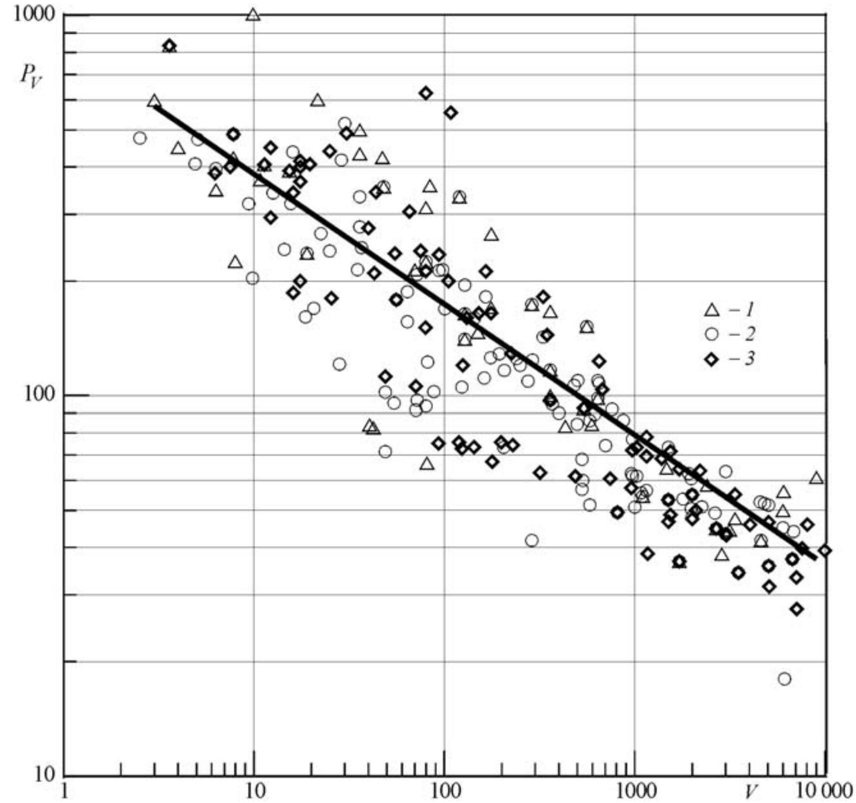

Fig. 1. Installed specific capacity of chamber electric resistance furnaces vs. workspace volume (logarithmic space): 1) T_{max} = 1150, 2) 1250, and 3) 1300°C. Statistics for 298 models of furnaces with working temperatures of 1150–1300°C of the main foreign manufacturers of such furnaces. P_V, W/liter; V, liter.

and inefficient, in our opinion. The second method is to mathematically model thermal operating regimes of an electric furnace and to select the capacity on the basis of modeling results.

It would seem that electric furnaces have been constructed for more than 100 years and all technical issues have long been dealt with and solved. However, the issue of selection of the electric-furnace capacity is far from being simple. Figure 1 compares the installed capacities for 298 models of industrial chamber electric resistance furnaces with working temperatures of 1150 to 1300°C from the main manufacturers of such equipment: "Nakal" (Russia), "Nabertherm" (Germany), "LAC" (Czech Republic), "Umega" (Lithuania), "SNOL" (Russia), "Uraleléktropech" (Russia), and "Kerammash" (Ukraine). The specific installed power per unit volume of the workspace is a characteristic convenient for comparison. As is seen from the figure, the installed capacity may differ by nearly an order of magnitude for furnaces with the same working temperature and the same workspace. For example, for furnaces with a maximum working temperature of 1300°C and a workspace volume of about 100 liters, the specific installed capacity varies from 75 to 600 W/liter (Fig. 1). We assume that the answer to the question of whether the value of the installed capacity is correct must be found with mathematical-modeling methods. It is precisely this issue that is the focus of the present paper.

Before we pass on directly to mathematical models, we make a few qualitative evaluations. An analysis of the characteristics of the above-mentioned furnaces (Fig. 1) from the viewpoint of the installed-capacity distribution reveals an interesting fact. The specific installed capacity obeys the approximately simple power law

$$P_V \sim V_{\text{fur}}^{-0.35\pm0.05} \sim V_{\text{fur}}^{-1/3} . \tag{1}$$

The energy in an electric resistance furnace goes mainly into the heating of a charge, warmup of the lining, and the loss from enclosing structures. The first two components are in proportion to the mass of the charge and the lining respectively, whereas the heat loss is in proportion to the area of the enclosing structures. Let the length l of the furnace's working chamber be the determining spatial scale. Also, we assume that the proportions of the furnace (width-to-length and height-to-length ratios) remain constant with variation in its dimensions. Then the lining mass will change as

$$M_{\mathrm{f}} \sim l^2 \delta \rho_{\mathrm{f}} \,. \tag{2}$$

Here δ and ρ_{f} are the lining thickness and density. Analogously the charge mass will be in proportion to the bottom area $M_{\mathrm{d}} \sim l^2$. In turn the external heat loss which is in proportion to the exterior-surface area will change as $P_{\mathrm{ext}} \sim l^2$. Since each of the three components of the energy expenditure in the furnace changes as its dimension squared, we can infer that the overall thermal power of the electric resistance furnace must grow as its linear dimension squared:

$$P_{\mathrm{fur}} \sim l^2 \,. \tag{3}$$

The volume of the furnace's working chamber is related to the linear dimension as $V_{\mathrm{fur}} \sim l^3$. Then, for the specific capacity of the furnace per unit volume of the workspace, expression (3) yields the relation

$$P_V = P_{\mathrm{fur}}/V \sim l^2/l^3 \sim l^{-1} \sim V_{\mathrm{fur}}^{-1/3} \,. \tag{4}$$

It is seen that our estimate (4) is consistent with relation (1), which has been obtained from an analysis of a wide range of electric-furnace models.

As has been noted above, the issues of thermal calculations of the parameters of electric resistance furnaces were the focus of quite an ample amount of literature [1–4]. The calculation methods presented in these works were developed in the 1960s–1970s and were intended for firebrick-lined furnaces. These methods were based on the use of numerous empirical coefficients and nomograms calculated precisely for firebrick lining. Unfortunately, these approaches cannot be applied to new furnaces with a lightweight fibrous lining whose thermophysical properties differ from the properties of fire brick by orders of magnitude. At the same time, a number of principles and approaches to designing electric resistance furnaces can be extended from firebrick-lined furnaces to fiber-lined ones. In this work, we present a mathematical model of thermal operation of an electric resistance furnace; this model makes it possible to calculate the characteristics of the furnace's thermal regime irrespective of the kind of materials used for lining and of the kind of heated products; also, the model uses the minimum amount of empirical information.

We consider a chamber electric resistance furnace with a parallelepiped-shaped workspace, which consists of the external skeleton, the lining, the heated products inside the furnace, and the electric heaters on the interior surface of the furnace walls. The heat-transfer model for this furnace must include: 1) the model of thermal-radiation transfer in the furnace's workspace, 2) the model of nonstationary heating of target products, 3) the model of nonstationary warmup of the furnace lining, and 4) the model of external heat exchange of the furnace's enclosing structures. Despite the general similarity of this model to the thermal model of a gas-flame furnace, they have a few fundamental differences, which will be considered below.

The final objective of construction of a mathematical model and of thermal calculation of the electric resistance furnace, which is based on this model, is formulated as follows: to calculate, from prescribed overall dimensions of the furnace, dimensions and properties of the lining, overall dimensions and properties of the heated parts, and from the temperature regime of their treatment, 1) the maximum thermal power required for electric-furnace operation, 2) the power-control ranges, and 3) all basic parameters of the electric heaters (material, dimension and shape, connection circuit, etc).

At temperatures in the workspace at 800–1000$^{\mathrm{o}}$C, the dominant role in heat exchange for electric resistance furnaces is played by radiative heat transfer. Therefore, description of radiative heat transfer must be the key part of the model. In this work, we use the two-dimensional model of radiative heat transfer, which has been presented in [5].

We formulate the basic approximations and assumptions of the general model:

1. Two-dimensional formulation of the problem of radiation transfer [5] and the problem of nonstationary heating of parts. Heat fluxes to the end surfaces of the furnace are allowed for in the approximation that the density of these fluxes is coincident with the density of the heat fluxes incident on the furnace's lateral walls.

2. Neglect of the energy expenditure in warming up the electric heaters.

3. Approximation of an isothermal heater: the absence of the temperature field inside the heaters with variation in the heater temperature with time.

4. Neglect of the convective component of heat exchange (we assume that there are no high-temperature fans in the furnace).

5. One-dimensional approximation for calculation of convective heat exchange on the exterior surfaces of the furnace.

If a furnace is not intended for chemical and heat treatment, its workspace is filled with air in which a small amount of optically active gases (primarily steam) is present in the infrared spectral region. It is proposed that the characteristics of radiant heat exchange in the furnace's working chamber be calculated by solution of the radiation-transfer equation. On condition of local thermodynamic equilibrium, this equation expresses the law of conservation of energy in its propagation in an absorbing radiating, and scattering medium. This equation has the following form [6]:

$$\mathbf{l}\cdot\nabla I_\lambda(\mathbf{r},\mathbf{l}) + [\chi_\lambda(\mathbf{r}) + \sigma_\lambda(\mathbf{r})]\, I_\lambda(\mathbf{r},\mathbf{l}) = \chi_\lambda(\mathbf{r})\, B_\lambda(T(\mathbf{r})) + \frac{\sigma_\lambda(\mathbf{r})}{4\pi}\int_{4\pi} p_\lambda(\mathbf{r},\mathbf{l},\mathbf{l}')\, I_\lambda(\mathbf{r},\mathbf{l}')\, d\Omega' , \tag{5}$$

where $B_\lambda(T) = c_1 n^2/[\pi\lambda^5(\exp(c_2/\lambda T) - 1]$ is the spectral intensity of blackbody radiation at temperature T in a medium with a refractive index n ($c_1 = 3.741832\cdot10^{-16}$ W·m^2 and $c_2 = 1.438786\cdot10^{-2}$ m·K). The boundary conditions to Eq. (5) are determined by the processes of radiation and reflection on the furnace's boundary surfaces:

$$I_\lambda(\mathrm{P},\mathbf{l}')_{(\mathbf{l}\cdot\mathbf{n})<0} = I_{0\lambda}(\mathrm{P},\mathbf{l}) + \frac{1}{\pi}\int_{2\pi} \rho_\lambda(\mathrm{P},\mathbf{l},\mathbf{l}')\, I_\lambda(\mathrm{P},\mathbf{l}')\,(\mathbf{l}'\cdot\mathbf{n})\, d\Omega' . \tag{6}$$

On the basis of the radiation-intensity field calculated from Eqs. (5) and (6), we determine the local densities of the resulting radiative flux $q_{\mathrm{w}}^{\mathrm{r}}(\mathrm{P})$ onto the heat-absorbing surfaces, which are necessary for solving the conjugate heat-exchange problem:

$$q_{\mathrm{w}}^{\mathrm{r}}(\mathrm{P}) = \int_0^\infty \varepsilon \left(\int_{2\pi} I_\lambda(\mathrm{P},\mathbf{l})\,(\mathbf{l}\cdot\mathbf{n})\, d\Omega - \pi B_\lambda(T_{\mathrm{w}}(\mathrm{P})) \right) d\lambda . \tag{7}$$

These equations must be solved with allowance for the specific geometry of the computational domain and for the optical properties of the gaseous medium in the infrared range.

For the parts treated (heated) in the furnace, the temperature is determined by solution of the unsteady two-dimensional equation of heat conduction without internal heat sources

$$c_p(T)\,\rho(T)\,\frac{\partial T(r,t)}{\partial t} = \nabla(\lambda(T)\,\nabla T(r,t)) , \tag{8}$$

where $r = (x, y)$ is the running coordinate of a point. Equation (8) must be supplemented with the initial and boundary conditions. The initial condition is as follows:

$$T(r,0) = T_{\mathrm{d}0} = \mathrm{const} . \tag{9}$$

At all boundaries of a part, we prescribe, at each instant of time, the resulting heat fluxes obtained from solution of the radiation-transfer problem:

$$q_s(\mathbf{r}, t) = -\lambda(T)\left.\frac{\partial T(\mathbf{r}, t)}{\partial \mathbf{n}}\right|_{\mathbf{r} \in W} = q_{\mathrm{res,d}}(\mathbf{r}) = \int_0^{\infty} \varepsilon_\lambda \left(-\int_{2\pi} I_\lambda(\mathbf{r}, \mathbf{l}) \cdot (\mathbf{l} \cdot \mathbf{n})\, d\Omega - \pi B_\lambda(T_{\mathrm{d}}(\mathbf{r})) \right) d\lambda\,, \tag{10}$$

where $\mathbf{n}$ is the external normal to the boundary W of a heated part and $q_{\mathrm{res,d}}(\mathbf{r})$ is the density of the resulting radiative flux onto the part's surface.

We note that the heat source in the resistance furnace is the hot surface of metallic heaters. These heaters are considered as being isothermal in the model; their temperature at each instant of time is found from the law of conservation of energy, which will be indicated below. To solve the radiation-transfer problem for the heaters we specify a boundary condition of the form (7).

A batch furnace is warmed up from a cold state. Therefore, the temperature of the furnace lining must be determined by solution of the nonstationary heat-conduction problem:

$$c_{p\mathrm{w}}(T_{\mathrm{w}})\, \rho_{\mathrm{w}}(T_{\mathrm{w}}) \frac{\partial T_{\mathrm{w}}(r, t)}{\partial t} = \nabla\left(\lambda_{\mathrm{w}}(T_{\mathrm{w}})\, \nabla T_{\mathrm{w}}(x, t)\right) \quad \text{(walls)}\,, \tag{11}$$

$$c_{p\mathrm{bot}}(T_{\mathrm{bot}})\, \rho_{\mathrm{bot}}(T_{\mathrm{bot}}) \frac{\partial T_{\mathrm{bot}}(y, t)}{\partial t} = \nabla\left(\lambda_{\mathrm{bot}}(T_{\mathrm{bot}})\, \nabla T_{\mathrm{bot}}(y, t)\right) \quad \text{(furnace bottom)}\,. \tag{12}$$

The initial condition for the furnace's enclosing structures must be specified irrespective of condition (9):

$$T_{\mathrm{w}}(r, 0) = T_{\mathrm{w},0} = \mathrm{const}\,. \tag{13}$$

The boundary heat-exchange conditions for the interior surfaces of the bottom and the walls are coincident with expression (10) in form. On the exterior surfaces, we specify the heat-exchange conditions of the third kind where the heat-exchange coefficients are described from the criterial relations (known from the literature) for free-convective heat exchange [7, 8]:

$$q_{\mathrm{w,ext}}(\mathbf{r}, t) = -\lambda(T)\left.\frac{\partial T_{\mathrm{w}}(\mathbf{r}, t)}{\partial \mathbf{n}}\right|_{\mathbf{r} \in W} = \alpha_{\mathrm{ext}}(T_{\mathrm{w,ext}} - T_0) + \varepsilon\sigma(T_{\mathrm{w,ext}}^4 - T_0^4)\,. \tag{14}$$

For the furnace roof, we can use the coefficient of free-convective heat exchange on a horizontal heat-release surface facing upward [8]:

$$\mathrm{Nu} = 0.15\, \mathrm{Ra}^{1/3}\,, \quad \mathrm{Ra} \in (8 \cdot 10^6\,, \;\; 3 \cdot 10^{10})\,, \tag{15}$$

$$\alpha_{\mathrm{ext}l} = 0.15 \lambda_{\mathrm{air}} \left[\frac{g\beta(T_{\mathrm{av}} - T_\infty)}{\nu_{\mathrm{air}} a} \right]^{1/3}. \tag{16}$$

Here $\mathrm{Nu} = \alpha_{\mathrm{ext}}/\lambda$ is the Nusselt number, $\mathrm{Ra} = g\beta l^3 (T_{\mathrm{av}} - T_\infty)/(\nu_{\mathrm{air}} a)$ is the Rayleigh number, g is the free-fall acceleration, β is the thermal coefficient of volumetric expansion (equal to $\beta = 1/T$ for ideal gases), T_{av} is the average temperature of the exterior roof surface, T_∞ is the ambient temperature, a is the thermal diffusivity of air, and ν_{air} is the coefficient of kinematic viscosity of air. All coefficients in (16) must be computed at the temperature $T = (T_{\mathrm{av}} + T_\infty)/2$; the characteristic dimension l is found as the ratio of the total area of the exterior surface of the roof to its perimeter.

Heat exchange for the exterior lateral walls of the furnace will be determined by the relation for the free-convective heat exchange on a vertical surface [8]:

$$\mathrm{Nu}^{1/2} = 0.825 + \frac{0.387\ \mathrm{Ra}^{1/6}}{\left[+ \left(\frac{0.492}{\mathrm{Pr}}\right)^{9/16}\right]^{8/27}} . \tag{17}$$

Heat transfer for the exterior surface of the furnace bottom will be described, in the approximation of free-convective heat exchange, as that for the heat-release horizontal surface facing downward:

$$\mathrm{Nu} = 3.88 + 0.0775\ \mathrm{Gr}^{1/3} . \tag{18}$$

The Grashof number in (18) is determined by the expression

$$\mathrm{Gr} = g\beta l^3 \left(T_{\mathrm{av}} - T_{\infty}\right)/\nu_{\mathrm{air}}^2 .$$

Thus, the given formulation of problem (4)–(18) is closed; it makes it possible to calculate the heat-engineering characteristics of the furnace from a prescribed geometry, the thermal and physical properties, the charge mass, and the temperature regime of heating.

A major feature of the given model is the method of its solution. The fact is that to solve the central problem — that of radiative heat transfer — we must know the temperatures of all surfaces inside the furnace, including the unknown temperature of the electric heaters. Either the heater capacity or the average temperature in the working chamber is prescribed according to the conditions of solution of such problems. Therefore, an additional problem arises: the heater temperature is not found without solution of the radiation-transfer problem, whereas the radiation-transfer problem is not solved if the heater temperature is unknown. To resolve this contradiction we propose, in this work, to use the law of conservation of energy as applied to an electric heater and to find the temperature of heaters satisfying this law using one of the most versatile methods of solution of nonlinear problems — the half-division method. The input parameters of the problem are the temperatures of all objects inside the furnace (except the heaters) and the capacity and surface area of the heaters. The surface temperatures of the parts and the lining are found either from the initial conditions (on the first time step) or by solution of the conjugate problem on the previous time step. Thus, two limiting points of the heating temperature, e.g., 100 and 1200°C, are prescribed. The total radiation-transfer problem is solved for these temperature, and the resulting heat fluxes radiated by the heaters are found. The key parameter for solution of the problem is the difference between the prescribed capacity of the heater and the resulting power received by all heat-absorbing surfaces in the furnace's working chamber. The resulting power is found by solution of the radiation-transfer problem. The main condition for successful solution is the heater temperature falling within the "fork." Thus, the above power difference must have different signs on the edges of the selected temperature interval. Next the interval is divided in two and the radiation transfer problem is solved once again with a new temperature. The obtained power difference is compared to the difference at the limiting points. A limiting point shifts to the new, middle point if the difference of the powers at them has the same sign. The final problem is reduced to finding such a heater temperature that will ensure, under running conditions, the fulfillment of the law of conservation of energy: the power radiated from the heater surface will be equal to the power released in it on traversal of an electric current. We note that to find the heater temperature accurate to 1°C in the above example with limiting points at 100 and 1200°C we must solve the radiation-transfer problem 12 times at each time step. This procedure is rather time-consuming as far as computations are concerned. But practice has shown that for today's computers, calculation of such problems takes from several tens of minutes to an hour or two (depending on the overall dimensions of the furnace, the duration of the heating regime, accuracy of calculations, etc.) Moreover, one can construct such numerical algorithms that will operate with a much narrower temperature "fork," which makes the calculations faster, too.

Several more features of the proposed procedure of solution of the model of the furnace's thermal regime are noteworthy. It is necessary to find a compromise version making it possible to simply describe such a geometrically intricate object as electric heaters (e.g., zigzag coils or coils on tubes). In this work, we propose, for this purpose, the approximation where intricately shaped heaters are replaced by an equivalent plate. In this case the equivalence condition is the equality of the surface areas of the heaters and the plate. As a substantiation of such an approximation, we note that the angular coefficients for a system of two bodies of a simple geometric shape remain virtually constant

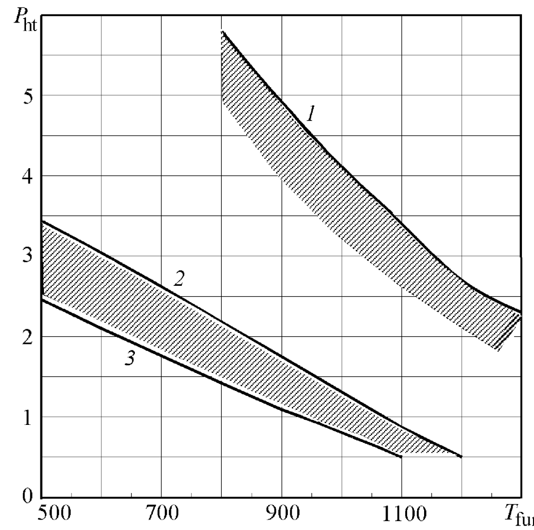

Fig. 2. Recommended specific surface power of high-resistance alloys used for manufacture of electric-resistance-furnace heaters: 1) Kanthal A-1 alloy (wire coiled onto ceramic tubes) [10]; 2 and 3) maximum and minimum recommended powers respectively for Kh20N80N (Nichrome) alloy [4]. P_{ht}, W/cm^2; T_{fur}, ^{o}C.

once the distance between them has become larger than two to three characteristic spatial scales (see, e.g., Appendix 3 in [9]). The coiling diameter offers such a scale for coil-type heaters. In other words, we assume in our model that at large distances compared to the dimension of an object, small-scale geometric features (irregularities) of its surface may not substantially affect the characteristics of radiative heat exchange. Therefore, the heater with an intricate geometry is replaced by a heater with a simple geometry with the same surface area.

The second important aspect is associated with selection of the heaters' area. As is well known, one of the most important characteristics of the heater material is the maximum permissible surface power (Fig. 2). It is the practice to measure it in W/cm^2. This characteristic substantially changes with increase in the temperature in the furnace. This characteristic of heaters must be taken into account when the furnace's thermal regime is calculated. Since the temperature of the heater will be dependent on its area, all other things being equal, in solving the problem on finding the maximum power of an electric furnace, the algorithm of solution is constructed as follows. Initially we make simple thermophysical evaluations (by the maximum permissible heating rate) of the maximum power of the furnace. Thereafter, with allowance for the properties of the selected heater material (Fig. 2), we evaluate the overall area of the heaters and their geometric dimensions. With these dimensions of the heaters we calculate the furnace's thermal regime. If the calculation yields that the maximum thermal power of the heaters significantly differs from the initial estimate, we evaluate the permissible heater area by this new value and repeat the calculation.

Thus, the above mathematical model and algorithm of numerical calculation make it possible to solve the problem on finding the characteristics of thermal regimes of electric resistance furnaces. We demonstrate this with an example.

The initial information for calculations performed below was taken from the technical assignment of one industrial enterprise. Therefore, the calculation results are close to practical needs as much as possible.

The calculations were performed for a large nonstandard chamber electric resistance furnace with workspace dimensions 1450 × 1450 × 2580 mm. The furnace was lined with fireclay brick. The lining thickness was 300 mm. The maximum mass of the charge together with the container was 1850 kg (steel and cast-iron parts). The rate of heating of the charge might not exceed 150^{o}C/h at temperatures above 600^{o}C. The furnace was intended for heat treatment of metal; therefore, the maximum temperature of the furnace required to heat the charge was 1000^{o}C. In this case we were confronted with the problem of modernization of the furnace. It was necessary to replace the firebrick lining by a fibrous one, with the overall dimensions of the furnace being preserved. Prior to modernization, the in-

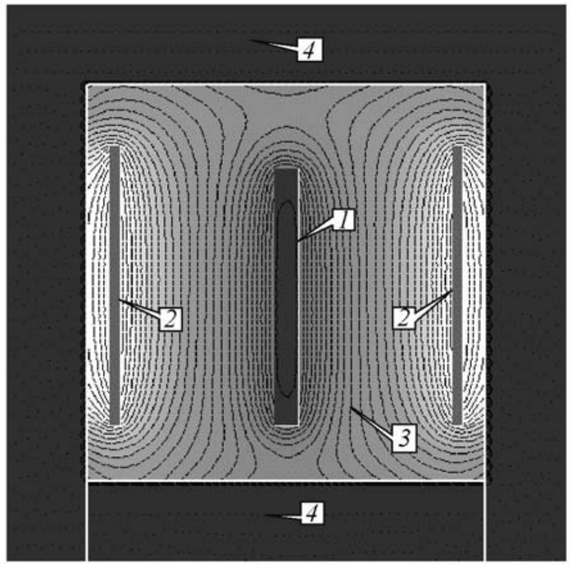

Fig. 3. Characteristic field (isolines) of thermal-radiation intensity in the furnace's workspace in the initial stage of warmup of the charge (two-dimensional cross section of the furnace's working chamber): 1) heated metal; 2) electric heaters; 3) furnace's workspace; 4) furnace's enclosing structures (lining).

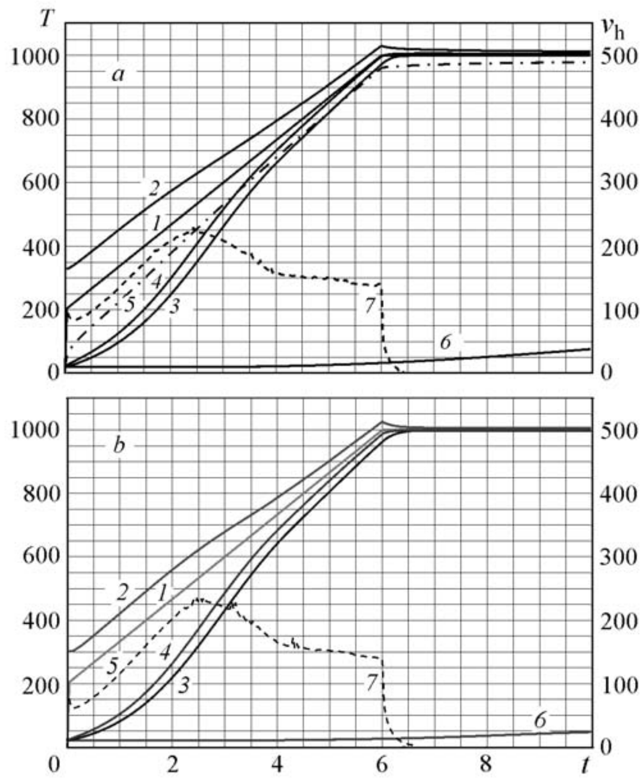

Fig. 4. Results of calculation of the thermal regime of the electric furnace lined with fireclay brick (a) and fibrous materials (b): 1) temperature in the furnace's workspace (prescribed regime of heating); 2) temperature of the electric heaters; 3 and 4) minimum and maximum temperature of the metal; 5 and 6) temperature of the interior and exterior lining surface; 7) rate of heating of the metal. T, $^{\circ}$C; t, h; v_h, $^{\circ}$C/h.

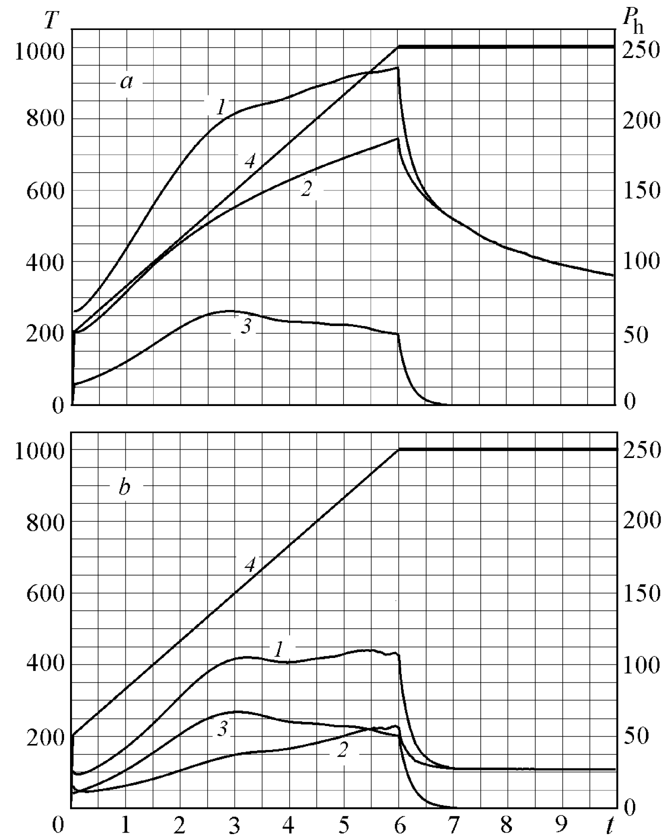

Fig. 5. Results of calculation of the thermal regime of the electric furnace lined with fireclay brick (a) and fibrous materials (b): 1) overall power necessary for maintaining a prescribed regime; 2) resulting thermal power incident on the furnace lining; 3) resulting thermal power incident on the metal; 4) temperature in the furnace's workspace (prescribed regime of heating). T, ^{o}C; t, h; P_{h}, kW.

stalled capacity of the furnace was 210 kW. It was necessary, due to the decrease in the lining mass, to diminish the installed capacity of the modernized furnace to a level that would ensure the above-described technological regime.

To evaluate the parameters of the furnace after modernization we performed its detailed thermal calculation from the solution of the conjugate heat-exchange problem presented above. We note that to somewhat simplify the computational procedure we concentrated the entire charge of the furnace in one part of a simple shape and arranged this part at the center of the workspace (Fig. 3). The thickness of the part corresponded to the characteristic thickness of parts treated in the furnace. The results of calculation of the characteristic field of thermal-radiation intensity in the initial stage of warmup of the charge for the problem in question are presented in the same figure.

In the calculations, we considered two versions: 1) the entire lining of the furnace is made of fireclay brick; 2) the lining of the lateral walls and roof is made from fibrous materials, whereas the lining of the bottom is a combination type: fibrous plates and fireclay brick.

The calculation results are presented in Figs. 4 and 5. Analyzing the modeling results, we note that the maximum power necessary for maintaining the considered regime in the old furnace with fireclay lining is 230 kW (Fig. 5a, curve 1), which is in good agreement with an actual installed capacity of 210 kW. This fact indirectly confirms the correctness of the performed calculations and the model. It can be seen from the data given in Fig. 5a that the basic power of the furnace goes into warming up the fireclay lining (curve 2). Also, we note that in the considered regime of heating, only 20% of the electric power is expended in heating the metal (with allowance for the holding time of the metal). The maximum power in heating the metal is reached at 500–700^{o}C (curve 3). The power necessary for warming up the furnace lining grows monotonically with increase in the temperature and begins to drop only when the holding zone has been attained.

On replacement of the firebrick lining by a fibrous one, the situation changes qualitatively (Fig. 5b). The heated metal, not the lining, constitutes the major portion in thermal balance now. The energy expenditure in warming up the lining diminishes 2.5–3 times (curves 2 in Fig. 5). The thermal efficiency of the furnace grows to 50%, i.e., 2.5 times, with allowance for the holding zone.

Thus, according to the results of computer modeling, the maximum thermal power of the modernized furnace, on replacement of the fireclay brick by a fibrous lining, is 110 kW for the prescribed overall dimensions of the furnace and the specified heating conditions. Since the accuracy of calculations is comparable in both cases, we can infer that replacement of the traditional fireclay brick by today's fibrous materials in batch electric resistance furnaces makes it possible to reduce the installed capacity of the furnaces by half and to increase the efficiency of such furnaces 2–2.5 times. Also, the modeling results make it possible to draw other interesting conclusions of practical importance. Thus, the replacement of the fireclay lining by a fibrous one will allow a reduction of 25^{o}C — from 65 to 40^{o}C — in the temperature of the exterior furnace surfaces, on the average. On replacement of the furnace lining, the specific energy expenditure in heat treating the metal must decrease 2.0–2.2 times for the considered regime of heating (see Fig. 4). The information obtained as a result of calculation enables us to calculate the parameters of heaters for the modernized furnace. Evaluation of these parameters has shown that the mass of Nichrome heaters can be diminished by 70 kg — from 200 to 130 kg — due to the decrease in the installed capacity of the electric furnace. With allowance for the high cost of high-resistance alloys and for the necessity of repairing the heaters periodically, this is a very substantial savings, too.

Thus, we can infer that the proposed mathematical model of thermal regime of an electric resistance furnace and the method of its numerical solution make it possible to find the basic characteristics of thermal operating regimes of such furnaces and are an important tool in modernizing the existing furnaces and designing new models of furnaces and in optimizing the structural parameters of the existing models of furnaces.

NOTATION

$B_\lambda(T)$, spectral intensity of black-body radiation at temperature T; c_p, heat capacity, J/(kg·K); $I_\lambda(\mathbf{r}, \mathbf{l})$, spectral radiation intensity at the point $\mathbf{r}$ in the direction $\mathbf{l}$; $I_{0\lambda}(\mathrm{P}, \mathbf{l})$, spectral intensity of the intrinsic radiation or the radiation transmitted from the outside at point P of the boundary; l, characteristic dimension of the furnace, m; M, mass, kg; $\mathbf{n}$, external normal to the boundary; P_{ext}, power of the external heat loss, W; P_V, specific installed capacity of the furnace, W/liter; P_{ht}, specific capacity (power) of the heater, W/cm^2; P_{h}, heating power, kW; $p_\lambda(\mathbf{r}, \mathbf{l}, \mathbf{l}')$, indicatrix of scattering of radiation in its interaction of the volume element of the medium; $q_{\mathrm{w}}^{\mathrm{r}}(\mathrm{P})$, local density of the resulting radiation flux onto the heat-absorbing surfaces at point P; $\mathbf{r}$, radius vector; T, temperature, ^{o}C; t, heating time, h; V_{fur}, volume of the furnace's workspace, liters; v_{h}, heating rate, ^{o}C/h; ε, emissivity factor of the surface; $\chi_\lambda(\mathbf{r})$ and $\sigma_\lambda(\mathbf{r})$, spectral coefficients of absorption and scattering respectively; λ, electromagnetic-radiation wavelength; $\rho_\lambda(\mathrm{P}, \mathbf{l}, \mathbf{l}')$, spectral coefficient of reflection of the boundary; ρ, density, kg/m^3; σ_0, Stefan–Boltzmann constant; Ω, solid angle. Subscripts: ext, exterior surface; fur, furnace; d, heated product; w, furnace wall; bot, furnace bottom; λ, spectral characteristic; ht, heater; h, heating; p, pressure; res, resulting; f, (fire-clay) lining; r, radiative; air, air; av, average; max, maximum; s, surface.

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